

N=2, D=6 supergravity with E_7 gauge matter

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Abstract

The lagrangian of N=2, D=6 supergravity coupled to $E_7 \times SU(2)$ vector- and hypermultiplets is derived. For this purpose the coset manifold $E_8/E_7 \times SU(2)$, parametrized by the scalars of the hypermultiplet, is constructed. A difference from the case of $Sp(n)$ -matter is pointed out. This model can be considered as an intermediate step in the compactification of D=10 supergravity coupled to $E_8 \times E_8$ matter to four-dimensional model of E_6 unification.

Introduction

Minimal six-dimensional supergravity has N=2 supersymmetries and can be coupled to the vector multiplet in adjoint representation of arbitrary gauge group. On the other hand, the hypermultiplet must belong to certain group to be coupled to the supergravity. In particular, the complete lagrangian with all couplings has been constructed in [1] for the case of $Sp(n) \times SU(2)$ gauge group. The group $SU(2)$ acts on two supersymmetry generators while $Sp(n)$ transforms only matter fields. The scalars of the hypermultiplet parametrize the coset manifold $Sp(n, 1)/Sp(n) \times SU(2)$, which possesses special properties, allowing one to construct the supersymmetric action.

Other coset manifolds were suggested in [1, 2] as candidates for the spaces the hypermatter could form. Of particular interest is the largest exceptional one, namely $E_8/E_7 \times SU(2)$. In this paper we explicitly construct this manifold and present the lagrangian of the N=2, D=6 supergravity coupled to the $E_7 \times SU(2)$ vector- and hyper-multiplets. In the rest of the Introduction we argue, that this model may play important role in the compactification of the $E_8 \times E_8$ heterotic string.

The compactification of D=10 supergravity, which is the low-energy limit of superstrings, to four-dimensional space-time is rather ambiguous [3]. The topological structure of the internal Calabi-Yau manifold is not determined. Moreover, complexity of six-dimensional spaces requires many unknown parameters to be introduced in order to obtain a predictable multigeneration model in four dimensions.

On the other hand, if one suggests, that at some intermediate energy-scale between the Plank mass and the Grand Unification scale the space-time is effectively six-dimensional, many problems get fixed. In this case internal four-dimensional manifold has $SU(2)$ holonomy group and, consequently, selfdual Ricci tensor; this is K_3 with necessity [4]. Moreover, the vacuum configuration guarantees the vanishing cosmological constant in six dimensions [5], even if higher-derivative corrections are included.

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After the compactification to six dimensions gauge vectors with four-dimensional index become scalars. They belong to $(248,1)+(1,248)$ representation of $E_8 \times E_8'$ gauge group and $4=(2,1)+(1,2)$ representation of $O(4) \sim SU(2) \times SU'(2)$ Lorentz group of internal manifold. Since $Tr_{E_8 \times E_8'} = Tr_{E_8} + Tr_{E_8'}$ one can expect that E_8' decouples. It is then natural [6] to pick up the singlet of $SU'(2) \times SU''(2)$, where $SU''(2)$ is a subgroup of E_8 . The residual group is $E_7 \times SU(2)$ and the scalars belong to its $(56, 2)$ representation. They can form the coset manifold mentioned above.

Nevertheless the compactification $N=1, D=10 \rightarrow N=2, D=6$ may turn out to be difficult to perform explicitly. In particular, the vector responsible for gauging of $SU(2)$ group is the ten-dimensional spin-connection $\hat{\omega}_m$, which is composite field, rather than independent degree of freedom. But since all couplings of six-dimensional supergravity are fixed uniquely up to two gauge coupling constants g and g' , it is interesting to construct the lagrangian independently. Possibly, the consistent consideration of the compactification may determine these constants too.

The compactification $N=2, D=6 \rightarrow N=1, D=4$ shouldn't be a problem since all two-dimensional equations of motion are explicitly solvable. The E_7 group is expected to be broken down to anomaly free E_6 , or further to $SO(10)$ group. One may hope that chiral compactification [7] on the sphere S^2 can solve the problem of mirror generations, which inevitably present in any real group such as E_7 or E_8 .

$E_8/E_7 \times SU(2)$ coset manifold

In order to construct the coset manifold $E_8/E_7 \times SU(2)$, we need E_8 algebra. For this purpose we use the $E_8 \supset E_7 \times SU(2)$ decomposition:

$$248 = (133, 1) + (56, 2) + (1, 3)$$

So the generators of E_8 are $(Y_\Sigma, I_s, Q_{i\alpha})$, where Y_Σ are E_7 generators, I_s are $SU(2)$ ones and $Q_{i\alpha}$ are off-diagonal coset generators. All index notations are given in Table 1. The E_8 commutation relations have the form [8]:

$$\begin{aligned} [Y_\Sigma, Y_\Lambda] &= f_{\Sigma\Lambda\Pi} Y_\Pi & [I_s, I_t] &= \varepsilon_{stu} I_u & [Y_\Sigma, I_s] &= 0 \\ [Y_\Sigma, Q_{i\alpha}] &= (T_\Sigma)^\beta{}_\alpha Q_{i\beta} & [I_s, Q_{i\alpha}] &= -\frac{i}{2} (\sigma_s)^j{}_i Q_{j\alpha} \end{aligned}$$

indices:	values:	representation:
Σ, Λ, Π	$1, \dots, 133$	E_7 adjoint
α, β, γ	$1, \dots, 56$	E_7 fundamental
s, t, u	$1, 2, 3$	$SU(2)$ adjoint
i, j, k	$1, 2$	$SU(2)$ fundamental

Table 1: Group indices

$$[Q_{i\alpha}, Q_{j\beta}] = -(CT_\Sigma)_{\alpha\beta}\varepsilon_{ij}Y_\Sigma + \frac{i}{2}C_{\alpha\beta}(\varepsilon\sigma_s)_{ij}I_s \quad (1)$$

Here $(T_\Sigma)^\alpha{}_\beta$ are antihermitean 56×56 E_7 matrices, normalized as $\text{tr}(T_\Sigma T_\Lambda) = -6\delta_{\Sigma\Lambda}$, $[T_\Sigma, T_\Lambda] = f_{\Sigma\Lambda\Pi}T_\Pi$, $f_{\Sigma\Lambda\Pi}$ are E_7 structure constants, $C_{\alpha\beta}$ is antisymmetric E_7 -invariant matrix, $C^{\alpha\beta}$ denotes $C^+ = C^{-1}$. $(\sigma_s)^i{}_j$ are Pauli matrices, $\varepsilon^{ij} = \varepsilon_{ij} = i\sigma_2$. The normalization of generators is chosen in such way, that E_8 Cartan-Killing metric is $-30(\delta_{\Sigma\Lambda}, \delta_{st}, C_{\alpha\beta}\varepsilon_{ij})$. Due to E_7 Fierz identity [9]

$$(T_\Sigma)^\gamma{}_\alpha(T_\Sigma)^\delta{}_\beta = -\frac{1}{4}\left(\delta_\alpha^\delta\delta_\beta^\gamma + C^{\gamma\delta}C_{\alpha\beta} + C^{\gamma\lambda}C^{\delta\sigma}d_{\alpha\beta\lambda\sigma}\right)$$

the relations (1) satisfy the Jacobi identities. $d_{\alpha\beta\lambda\sigma}$ is totally symmetric E_7 -invariant tensor.

The scalars of the hypermultiplet $\Phi^{i\alpha}$ are the coordinates of the coset manifold. They satisfy the reality condition:

$$\bar{\Phi}_{i\alpha} \equiv (\Phi^{i\alpha})^* = \varepsilon_{ij}C_{\alpha\beta}\Phi^{j\beta}$$

The vielbein $V^{i\alpha}$ and spin-connections Ω^Σ, Ω^s of the coset manifold can be constructed by means of the Maurier-Cartan form (for differential geometry of coset manifolds see, for example, [10]):

$$L^{-1}\frac{\partial}{\partial\Phi^{i\alpha}}L = 2iV_{\underline{i\alpha}}{}^{j\beta}Q_{j\beta} + \Omega_{\underline{i\alpha}}{}^\Sigma Y_\Sigma + \Omega_{\underline{i\alpha}}{}^s I_s$$

$$\text{where } L = \exp\left(2i\Phi^{i\alpha}Q_{i\alpha}\right) \quad \text{is the coset representative,} \quad (2)$$

underlying index $\underline{i\alpha}$ is curved one. The multiplier $2i$ is introduced for conventional normalization of the field Φ in the lagrangian. Evaluating equation (2) we obtain the following expressions for the vielbein and spin-connections:

$$V_{\underline{i\alpha}}{}^{j\beta} = \left(\frac{\text{sh}\sqrt{M}}{\sqrt{M}}\right)^{j\beta}{}_{i\alpha}$$

$$\Omega_{\underline{i\alpha}}{}^\Sigma = -4\left(\bar{\Phi}T_\Sigma\frac{\text{ch}\sqrt{M}-1}{M}\right)_{i\alpha} \quad \Omega_{\underline{i\alpha}}{}^s = -2i\left(\bar{\Phi}\sigma_s\frac{\text{ch}\sqrt{M}-1}{M}\right)_{i\alpha} \quad (3)$$

where M is hermitean 112×112 matrix:

$$M^{j\beta}{}_{i\alpha} = -4(T_\Sigma\Phi)^{j\beta}(\bar{\Phi}T_\Sigma)_{i\alpha} + (\sigma_s\Phi)^{j\beta}(\bar{\Phi}\sigma_s)_{i\alpha} \quad (4)$$

In particular $M^{i\alpha}{}_{j\beta}\Phi^{j\beta} = 0$. Consequently the matrix M is degenerate and doesn't have inverse one, so the equations (3) should be considered as formal expansions in powers of M , which are positive. Due to the properties

$$\begin{aligned} \partial_{\underline{k\gamma}}f(M)^{j\beta}{}_{i\alpha}(T_\Sigma\Phi)^{k\gamma} &= [T_\Sigma, f(M)]^{j\beta}{}_{i\alpha} \\ \partial_{\underline{k\gamma}}f(M)^{j\beta}{}_{i\alpha}(\sigma_s\Phi)^{k\gamma} &= [\sigma_s, f(M)]^{j\beta}{}_{i\alpha} \end{aligned} \quad (5)$$

any function $f(M)$ transforms uniformly under $E_7 \times SU(2)$ transformations:

$$\delta\Phi = U^\Sigma T_\Sigma\Phi - \frac{i}{2}U^s\sigma_s\Phi$$

Furthermore, the calculation of $\partial_{[i\alpha}(L^{-1}\partial_{j\beta]}L)$ gives the following equations on the derivatives of the vielbein and spin-connections:

$$\begin{aligned} \partial_{[i\alpha}V_{j\beta]}^{k\gamma} + (T_\Sigma)^\gamma{}_\delta\Omega_{[i\alpha}^\Sigma V_{j\beta]}^{k\delta} - \frac{i}{2}(\sigma_s)^k{}_l\Omega_{[i\alpha}^s V_{j\beta]}^{l\gamma} &= 0 \\ \partial_{[i\alpha}\Omega_{j\beta]}^\Sigma + \frac{1}{2}f_{\Sigma\Lambda\Pi}\Omega_{i\alpha}^\Lambda\Omega_{j\beta}^\Pi + 2(C T_\Sigma)_{\gamma\delta}\varepsilon_{kl}V_{i\alpha}^{k\gamma}V_{j\beta}^{l\delta} &= 0 \\ \partial_{[i\alpha}\Omega_{j\beta]}^s + \frac{1}{2}\varepsilon_{stu}\Omega_{i\alpha}^t\Omega_{j\beta}^u - iC_{\gamma\delta}(\varepsilon\sigma_s)_{kl}V_{i\alpha}^{k\gamma}V_{j\beta}^{l\delta} &= 0 \end{aligned} \quad (6)$$

We will use the properties (5), (6) evaluating the supersymmetry algebra.

Coupling to supergravity

We construct the lagrangian following Noether procedure. One starts with the lagrangian of pure N=2, D=6 supergravity and adds terms, necessary to cancel its variation with respect to the supersymmetry transformations, modified due to the presence of the matter fields. The algebra is very similar to the $Sp(n)$ case [1], so we omit details of the calculation. We consider dual version of the N=2, D=6 supergravity, so that the field-strength tensor $B_{mnp} = 3\partial_{[m}B_{np]}$ does not contain Chern-Simons term. The lagrangian of the dual supergravity coupled to $Sp(n)$ -matter can be found in [11]. For simplicity we omit fourth-order fermionic terms in the lagrangian and third-order fermionic terms in the supersymmetry transformations of the fermions.

Letters a, b, c are used as flat space-time indices and m, n, p as curved ones. The metric and antisymmetric tensors are $\eta_{ab} = (+, -, \dots, -)$, $\varepsilon^{01\dots 5} = 1$. We use 8-component spinors and 8×8 Dirac matrices γ^a . Spinorial indices are not written explicitly in the text. As usual, $\gamma^{a_1\dots a_n}$ denotes antisymmetrized product of n γ -matrices. Notations of all fields and their conjugation rules are given in Table 2. The vielbein $V_{i\alpha}^{j\beta}$ is inverse to $V_{i\alpha}^{j\beta}$ one and $g_{i\alpha,j\beta} = \varepsilon_{kl}C_{\gamma\delta}V_{i\alpha}^{k\gamma}V_{j\beta}^{l\delta}$ is the metric of the coset manifold.

At first let us write down the definitions of the covariant derivatives for all fields:

$$\begin{aligned} D_m\Phi &= \partial_m\Phi - A_m^\Sigma T_\Sigma\Phi + \frac{i}{2}\mathcal{A}_m^s\sigma_s\Phi \\ D_m\Psi &= \Psi_{;m} - A_m^\Sigma T_\Sigma\Psi + D_m\Phi^{i\alpha}\Omega_{i\alpha}^\Sigma T_\Sigma\Psi \\ D_m\epsilon &= \epsilon_{;m} + \frac{i}{2}\mathcal{A}_m^s\sigma_s\epsilon - \frac{i}{2}D_m\Phi^{i\alpha}\Omega_{i\alpha}^s\sigma_s\epsilon \quad \text{the same for } \psi_m, \chi \\ D_m\lambda^\Sigma &= \lambda_{;m}^\Sigma - f_{\Sigma\Lambda\Pi}A_m^\Lambda\lambda^\Pi + \frac{i}{2}\mathcal{A}_m^s\sigma_s\lambda^\Sigma - \frac{i}{2}D_m\Phi^{i\alpha}\Omega_{i\alpha}^s\sigma_s\lambda^\Sigma \\ D_m\rho^s &= \rho_{;m}^s - \varepsilon_{stu}\mathcal{A}_m^t\rho^u + \frac{i}{2}\mathcal{A}_m^t\sigma_t\rho^s - \frac{i}{2}D_m\Phi^{i\alpha}\Omega_{i\alpha}^t\sigma_t\rho^s \end{aligned} \quad (7)$$

The semicolon denotes usual space-time covariant derivative; ϵ is the parameter of the supersymmetry transformations. The field-strength tensors are:

$$\begin{aligned} F_{mn}^\Sigma &= 2\partial_{[m}A_{n]}^\Sigma - f_{\Sigma\Lambda\Pi}A_m^\Lambda A_n^\Pi \\ \mathcal{F}_{mn}^s &= 2\partial_{[m}\mathcal{A}_{n]}^s - \varepsilon_{stu}\mathcal{A}_m^t\mathcal{A}_n^u \end{aligned} \quad (8)$$

multiplet:	fields:	notations:	$E_7 \times SU(2)$ rep.:	reality:
gravitational + antisymmetric:	graviton	e_m^a	(1,1)	real
	antisym. tensor	B_{mn}	(1,1)	real
	dilaton	φ	(1,1)	real
	gravitino	ψ_m^i	(1,2)	$\tilde{\psi}_{mi} = i\varepsilon_{ij}\psi_m^j$
	dilatino	χ^i	(1,2)	$\tilde{\chi}_i = -i\varepsilon_{ij}\chi^j$
hypermultiplet:	scalar	$\Phi^{i\alpha}$	(56,2)	$\Phi_{i\alpha} = \varepsilon_{ij}C_{\alpha\beta}\Phi^{j\beta}$
	fermion	Ψ^α	(56,1)	$\tilde{\Psi}_\alpha = -iC_{\alpha\beta}\Psi^\beta$
E_7 vector:	vector	A_m^Σ	(133,1)	real
	fermion	$\lambda^{\Sigma i}$	(133,2)	$\tilde{\lambda}_i^\Sigma = i\varepsilon_{ij}\lambda^{\Sigma j}$
$SU(2)$ vector:	vector	\mathcal{A}_m^s	(1,3)	real
	fermion	ρ^{si}	(1,3 \times 2)	$\tilde{\rho}_i^s = i\varepsilon_{ij}\rho^{sj}$

Table 2: N=2, D=6 supersymmetric multiplets. For all spinors $\tilde{\psi} = \bar{\psi}\mathcal{C}^+$, $\bar{\psi} = \psi^*\gamma^0$, \mathcal{C} is unitary symmetric charge conjugation matrix. Dilatino and Ψ are right-handed spinors $\gamma^7\chi = -\chi$, all other fermions are left-handed.

According to [1], the derivatives of the fields, belonging to fundamental representations of $E_7 \times SU(2)$ get modified by adding the terms with the spin-connections Ω . So the commutator becomes:

$$D_{[m}D_{n]}\epsilon = -\frac{1}{8}R_{mncd}\gamma^{cd}\epsilon + \frac{i}{4}\mathcal{F}_{mn}{}^s\sigma_s\epsilon + \frac{i}{4}\sigma_s\epsilon\Omega_{i\alpha}{}^s\left(F_{mn}^\Sigma T_\Sigma\Phi - \frac{i}{2}\mathcal{F}_{mn}^s\sigma_s\Phi\right)^{i\alpha} + \frac{1}{2}\sigma_s\epsilon D_m\Phi^{i\alpha}D_n\Phi^{j\beta}V_{i\alpha}{}^{k\gamma}V_{j\beta}{}^{l\delta}C_{\gamma\delta}(\varepsilon\sigma_s)_{kl}$$

We used properties (5), (6) proving this.

The lagrangian of the N=2, D=6 dual supergravity coupled with $E_7 \times SU(2)$ vector multiplets and hypermultiplet has the following form:

$$\begin{aligned}
e^{-1}L = & \frac{1}{4}R + \varphi_{;a}\varphi^{;a} + \frac{1}{12}e^{4\varphi}B_{abc}B^{abc} - \frac{i}{2}\bar{\psi}_a\gamma^{abc}D_b\psi_c + \frac{i}{2}\bar{\chi}\hat{D}\chi \\
& - i\varphi_{;a}\bar{\psi}_b\gamma^a\gamma^b\chi + \frac{i}{24}e^{2\varphi}\left(\bar{\psi}_a\gamma^{[a}\hat{B}\gamma^{b]}\psi_b + 2\bar{\psi}_a\hat{B}\gamma^a\chi + \bar{\chi}\hat{B}\chi - \bar{\Psi}\hat{B}\Psi\right) \\
& + \frac{1}{2}g_{i\alpha,j\beta}D_a\Phi^{i\alpha}D^a\Phi^{j\beta} + \frac{i}{2}\bar{\Psi}\hat{D}\Psi - iD_a\Phi^{i\alpha}V_{i\alpha}{}^{j\beta}\varepsilon_{jk}(\bar{\Psi}_\beta\gamma^b\gamma^a\psi_b^k) \\
& + \frac{1}{g^2}\left[-\frac{1}{4}e^{-2\varphi}F_{ab}^\Sigma F^{\Sigma ab} + \frac{1}{8}\varepsilon^{a_1\dots a_6}F_{a_1a_2}^\Sigma F_{a_3a_4}^\Sigma B_{a_5a_6} + ie^{-2\varphi}\bar{\lambda}^\Sigma\hat{D}\lambda^\Sigma\right. \\
& + \frac{i}{2}e^{-2\varphi}\bar{\lambda}^\Sigma\left(\gamma^a\hat{F}\psi_a + \hat{F}\chi\right)^\Sigma + \frac{i}{12}\bar{\lambda}^\Sigma\hat{B}\lambda^\Sigma\left. - 2i(T_\Sigma\Phi)^{i\alpha}V_{i\alpha}{}^{j\beta}\varepsilon_{jk}(\bar{\Psi}_\beta\lambda^{\Sigma i})\right. \\
& + \frac{1}{g^2}\left[-\frac{1}{4}e^{-2\varphi}\mathcal{F}_{ab}^s\mathcal{F}^{s ab} + \frac{1}{8}\varepsilon^{a_1\dots a_6}\mathcal{F}_{a_1a_2}^s\mathcal{F}_{a_3a_4}^s B_{a_5a_6} + ie^{-2\varphi}\bar{\rho}^s\hat{D}\rho^s\right. \\
& + \frac{i}{2}e^{-2\varphi}\bar{\rho}^s\left(\gamma^a\hat{\mathcal{F}}\psi_a + \hat{\mathcal{F}}\chi\right)^s + \frac{i}{12}\bar{\rho}^s\hat{B}\rho^s\left. - (\sigma_s\Phi)^{i\alpha}V_{i\alpha}{}^{j\beta}\varepsilon_{jk}(\bar{\Psi}_\beta\rho^{si})\right. \\
& + \frac{1}{2}(\bar{\psi}_a\gamma^a + \bar{\chi})\sigma_s(\lambda^\Sigma C^{\Sigma s} + \rho^t C^{ts}) - \frac{1}{8}e^{2\varphi}(g^2C^{\Sigma s}C^{\Sigma s} + g'^2C^{st}C^{st}) \quad (9)
\end{aligned}$$

where $\hat{B} = B^{abc}\gamma_{abc}$ and so on; g, g' are E_7 and $SU(2)$ coupling constants respectively. Following [1], we introduced real functions

$$C^{\Sigma s} = -2i \bar{\Phi} \sigma_s \frac{\text{ch}\sqrt{M} - 1}{M} T_\Sigma \Phi \quad C^{st} = \delta^{st} - \bar{\Phi} \sigma_t \frac{\text{ch}\sqrt{M} - 1}{M} \sigma_s \Phi$$

for notational convenience. For the same reason indices, labeling representations 2 and 56 are suppressed in those places, where their position can be restored unambiguously.

The lagrangian (9) is invariant with respect to the supersymmetry transformations, written below:

$$\begin{aligned} \delta e_m^a &= i \bar{\psi}_m \gamma^a \epsilon \\ \delta \varphi &= -\frac{i}{2} \bar{\chi} \epsilon \\ \delta B_{mn} &= i e^{-2\varphi} \left(-\bar{\psi}_{[m} \gamma_{n]} \epsilon - \frac{1}{2} \bar{\chi} \gamma_{mn} \epsilon \right) \\ \delta \psi_m &= D_m \epsilon - \frac{1}{24} e^{2\varphi} \hat{B} \gamma_m \epsilon \\ \delta \chi &= -\varphi_{;a} \gamma^a \epsilon - \frac{1}{12} e^{2\varphi} \hat{B} \epsilon \\ \delta \Phi^{i\alpha} &= i V_{j\beta}^{i\alpha} C^{\beta\gamma} (\bar{\Psi}_\gamma \epsilon^j) \\ \delta \Psi^\alpha &= D_a \Phi^{i\beta} V_{i\beta}^{j\alpha} \varepsilon_{jk} \gamma^a \epsilon^k \\ \delta A_m^\Sigma &= i \bar{\lambda}^\Sigma \gamma_m \epsilon \\ \delta \lambda^\Sigma &= -\frac{1}{4} \hat{F}^\Sigma \epsilon + \frac{i}{4} g^2 e^{2\varphi} C^{\Sigma s} \sigma_s \epsilon \\ \delta \mathcal{A}_m^s &= i \bar{\rho}^s \gamma_m \epsilon \\ \delta \rho^s &= -\frac{1}{4} \hat{\mathcal{F}}^s \epsilon + \frac{i}{4} g'^2 e^{2\varphi} C^{st} \sigma_t \epsilon \end{aligned} \tag{10}$$

The lagrangian (9) has the same form as in the case of $Sp(n)$ -matter [11]. Nevertheless there is one essential difference. In our case there is no need to impose the constraint $\varepsilon_{kl} V_{(i\alpha}^{k\gamma} V_{j\beta)}^{l\delta} = -n^{-1} g_{i\alpha, j\beta} C^{\gamma\delta}$ on the vielbein. This constraint has been introduced in [2] (eq.(2)) and used in [1, 11]. We never used this constraint checking the invariance of the lagrangian (9) with respect to the transformations (10). Moreover, the vielbein (3) does not satisfy this constraint. It can be easily seen in the point $\Phi = 0$ where $V_{i\alpha}^{j\beta} = \delta_i^j \delta_\alpha^\beta$.

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